convective transport is much longer than the time of the chemical reaction. Thus, τ_* is slightly dependent on the reaction time, which in turn is determined by the initial temperature.

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TWO-PHASE FLOW IN A CHANNEL WITH ERODING WALLS

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The problem of two-phase flow in an axisymmetric channel, the walls of which can be destroyed through erosion, is considered in the paper. Two-phase flows in channels accompanied by wall erosion were studied in [1-4]. Since the mechanism of energy transfer to the eroding surface can differ depending on the conditions at the flow boundaries, each of these investigations is of independent interest. Here we consider erosion under the impact action of solid or liquid particles [5], while the main mechanism of energy transfer is assumed to be convective transport of the condensed phase. An erosion model describing this case was proposed in [6], and it lay at the basis of the present work. Certain exact relations connecting the parameters of the condensed phase with the parameters of the profile of the axisymmetric channel are obtained. It is shown that the process of erosion with convective transport of particles to the wall develops unstably, and the time of development of the instability is estimated. A numerical quasi-one-dimensional model of two-phase flow in a channel with eroding walls is developed, permitting a wide range of parametric research.

1. Determination of Particle Trajectories in Two-Phase Flow

We consider the flow of a mixture of a gas and solid particles in an axisymmetric channel with curved walls. We take the gas to be inviscid and not thermally conducting in its interaction with the solid boundary and we neglect the intrinsic volume of the particles. To describe the flow we introduce a cylindrical coordinate system with the origin in the entrance cross section of the channel and we designate the axial coordinate as x and the radial coordinate as y. The following equations [7] are satisfied for the steady motion of particles of radius a along the trajectory of motion $y_a(x)$:

$$u_a du_a/dx = \varphi_a(u - u_a),\ u_a dv_a/dx = \varphi_a(v - v_a),$$

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$$dy_a/dx = v_a/u_a, \tag{1.1}$$

where u_a and v_a are the axial and radial components of the particle velocity, respectively; φ_a is a parameter of the force interaction between phases. By differentiating the last of Eqs. (1.1) and combining the result with the first two, we obtain the equation

$$l_a y_a'' + y_a' - \frac{v}{u} = 0. \tag{1.2}$$

Here the parameter $l_a = u_a^2/(\varphi_a u)$ has the dimension of length and is a measure of the difference of the particle trajectories from the streamlines of the carrier gas. In particular, as $l_a \to 0$ we get $y_a' = v/u$ from (1.2), i.e., the particle trajectories coincide with the streamlines of the carrier gas in this case; conversely, as $l_a \to \infty$ we have $y_a'' = 0$, i.e., the particles are free and move along straight lines in this case. For an ideal gas the condition of non-penetration ($u/v = y_w'$) is satisfied at the solid boundary $y_w(x)$ of the flow. Using the continuity equation, this condition can be extended into the interior of the flow region. Then near the wall we will have

$$v/u = y_w' + (y_w - y) \frac{d}{dx} \ln (\rho u y)_w + \ldots,$$

where the index w means that the indicated quantity is taken at the channel wall, while the dots denote terms of higher-order smallness in the expansion of the ratio v/u. Substituting this expression into Eq. (1.2), we finally have

$$l_a y_a'' + y_a' - y_w' + (y_a - y_w) \frac{d}{dx} \ln (\rho u y)_w = 0. \tag{1.3}$$

Equation (1.3) describes a bundle of trajectories of particles moving near the channel wall and making the main contribution to the energy flux to the surface being eroded. It was established earlier that in one case the transport of particles to the wall is due to the curvature of the streamlines of the carrier gas, while in another case everything is determined by the conditions at the channel entrance and by the greater inertia of the condensed phase [1-4]. Evidently, both cases can be important in practice, but in the present paper we shall consider only the first of them, which is described by Eq. (1.3).

Let us estimate the numerical value of l_a in Eq. (1.3). For a spherical particle we have

$$l_a \leqslant 8\rho_s u_a^2 a/(3\rho u \,|u - u_a|\, C_D),$$

where C_D is the drag coefficient; ρ_s is the density of the particle material. Taking $u_a = 0.7u$ and $C_D \approx 1$, which corresponds to motion of the particles with a considerable velocity lag, for $\rho_s/\rho = 4 \cdot 10^3$ we obtain the estimate $l_a \leqslant 2 \cdot 10^4\, a$. For micron-sized particles moving in a channel with a characteristic size $L \approx 1$ m we will have a small parameter $\varepsilon = l_a/L \leqslant 10^{-1}$ to the leading derivative in Eq. (1.3). The presence of the small parameter simplifies the investigation of the problem, which is a singularly perturbed problem [8, 9], however, imposing certain restrictions on the class of functions $y_w(x)$ figuring in the analysis. Using perturbation theory, we can obtain the following result: The inclusion of the particle trajectories as they strike the channel wall is determined from the equation (cf. with [9])

$$y_a' = y_w' - l_a y_w'' + O(\varepsilon^2), \quad \varepsilon \ll 1. \tag{1.4}$$

Here the local radius of curvature is chosen as the characteristic size. An analysis of Eq. (1.4) allows us to conclude that particles strike the wall in a section of flow where the profile has a negative curvature. Through perturbation theory we can also establish the point of incidence of a particle which began moving at the point with the coordinates (x_0, y_0). For the particle trajectory in the first approximation we have, from Eq. (1.3), the expression

$$y_a = y_g - (\rho u y)_w^{-1} \int_{x_0}^{x} (\rho u y)_w\, l_a y_w'' dx + O(\varepsilon^2),$$

where y_g is the gas streamline satisfying the condition $y_g(x_0) = y_0$. In particular, a trajectory exists for which the equality $y_g(x_0) = y_w(x_0)$ is valid, and therefore $y_g(x) = y_w(x)$. This line separates the region of pure gas near the wall from the region of two-phase flow. The separation of particles is usually observed in nozzles and is a peculiarity of two-phase flows [9]. The equation of the interface has the form

$$y_a^* = y_w - (\rho u y)_w^{-1} \int_{x_0}^{x} (\rho u y)_w\, l_a y_w'' dx + O(\varepsilon^2). \tag{1.5}$$

(We note that if $(\rho uy)_w \neq \text{const}$, then, contrary to [9], the equation of the dividing line (1.3) is reduced to (1.4) only for $y_a = y_w$.) Assuming that $y_a^*(x_1) = y_w(x_1)$ at a certain point x_1, from (1.5) we obtain the condition for particles striking the nozzle wall in the supersonic part in the form

$$\int_{x_0}^{x_1} (\rho uy)_w l_a y_w'' dx = (\rho uy)_w O(\varepsilon^2). \tag{1.6}$$

From (1.6) it follows that to find the functions $x_0(a)$ and $x_1(a)$ one must investigate the second order of perturbation theory for the solution of Eqs. (1.3). Thus, to within $\sim\varepsilon$ the coordinates $x_0(a)$, $x_1(a)$ where particles strike the wall do not depend on the particle size. Numerical calculations of particle trajectories confirm these conclusions. The relative quantity $(y_w - y_a^*)/y_w$ for the limiting trajectories is shown in Fig. 1: curves 1-3 are for particles with diameters of 1.5, 2.8, and 4.0 µm, respectively. The general form of the nozzle profile and one of the particle trajectories are shown in the upper part of Fig. 1. One can see that the smallest particles lie closer to the wall everywhere except for the erosion region in the supersonic part of the nozzle, where the largest particles (curves 2 and 3) strike before the small ones (curve 1). From an analysis of the limiting cases of $l_a \to 0, \infty$ for the solution of Eq. (1.3) it follows that the dependence $x_1(a)$ of the point of incidence on the particle size has a nonmonotonic character. This is confirmed by the numerical calculations. The dependence $x_1(a)$ is plotted in Fig. 2 (curve 1) for a nozzle with a parabolic generating line in the end section. A clear minimum in the $x_1(a)$ curve is observed here.

Usually the point x_0 corresponds to the part of the profile in the subsonic region of flow where the second derivative y_w'' of the profile changes sign (possibly with a jump). In the throat section of the nozzle, where $y_w'' > 0$, the integral (1.6) grows, so that a necessary condition for outward transport of particles in the supersonic part consists in the presence of a section of profile with a negative curvature, $y_w'' < 0$. Thus, in a conical nozzle $y_w'' = 0$ and the outward transport of particles is not possible here, as was established in [10] by a numerical analysis. On the other hand, $y_w'' < 0$ in a nozzle with a parabolic generating line, so that the transport of particles to the wall can occur. Another necessary condition for transport for a nozzle of finite length L consists in the presence of particle fractions whose size satisfies the condition $x_1(a) < L$.

Thus, the acceleration produced by the curvature of the streamlines of the carrier gas and acting on the inertial particles is the main cause of particle transport to the boundary of the two-phase flow in a channel of sufficiently large size. If the relaxation parameter l_a is larger ($l_a \gtrsim L$), however, then particle transport to the wall will be determined by the conditions at the channel entrance, mainly the initial inclination $y_a'(0)$ of the particle trajectories [4].

2. Model of Erosional Destruction

The particles reaching the channel walls have a considerable store of kinetic energy. A particle's velocity may exceed 1 km/sec when it strikes the supersonic part of the nozzle. In a high-velocity collision considerable stresses develop at the point of contact and the destruction of wall material occurs. The destruction effect becomes even more pronounced in the collision with a flux of particles. In this case the mass removal of wall materials occurs, which can be described mathematically by the function $y_w = y_w(x, t)$. According to [6], for the function $y_w(x, t)$ we have the equation (everywhere below we take $y_a' > y_w'$, otherwise we take $\partial y_w/\partial t = 0$)

$$\rho^* \frac{\partial y_w}{\partial t} = \sum_a E_a \rho_a u_a (y_a' - y_w'). \tag{2.1}$$

Here ρ^* is the density of the material being eroded (the wall); E_a is the erosion coefficient, which is a function of the parameters of the collision; ρ_a is the flux density of the discrete phase. For flow with monodisperse particles it is sufficient to retain one term on the right side of (2.1). An analysis of numerous data on erosion [2, 5, 11] and of data on high-velocity impact [12] leads to the conclusion that in the region of low collision angles the erosion coefficient can be represented in the form

$$E_a = (V_a^2/\sigma_{\text{er}})(\sin\alpha_a)^q, \quad \alpha_a \leqslant 20^\circ,$$

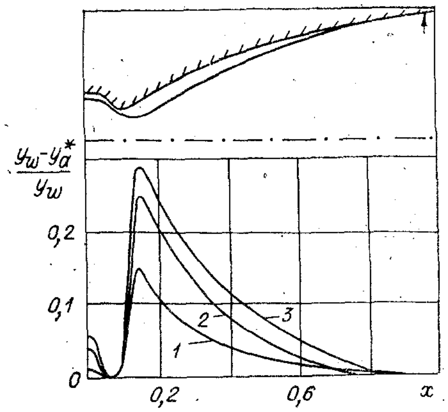

Fig. 1

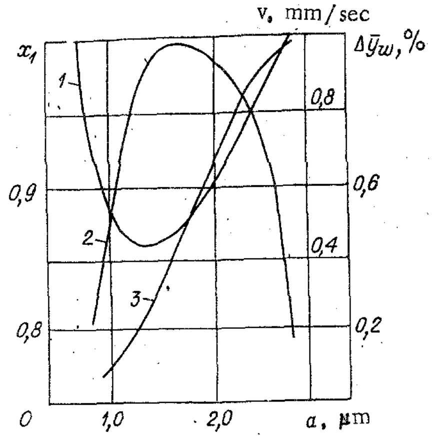

Fig. 2

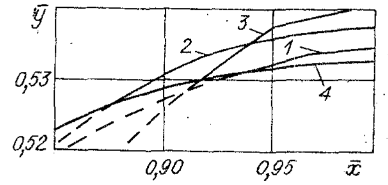

Fig. 3

where σ_{er} is a parameter characterizing the resistance of the material to erosional destruction; q is an exponent which can vary within wide limits for different materials. Thus, according to [11], q = 0.802 for graphite and q = 1.63 for glass at a moderate collision velocity. It is convenient to express the collision angle through the inclination of the trajectory and the local inclination of the wall through the equation

$$\alpha_a = \operatorname{arctg}\left(\frac{y'_a - y'_w}{1 + y'_a y'_w}\right)$$

and, in addition, to set $V_a^2 = u_a^2(1 + y_a'^2)^{q+1}$.

Then for erosion by monodisperse particles the equation of wall erosion will have the form

$$\frac{\partial y_w}{\partial t} = G\frac{1 + y_a'^2}{(1 + y'_a y'_w)^q}(y'_a - y'_w)^{q+1}, \tag{2.2}$$

where G is the characteristic erosion rate ($G = \rho u_a^3/(\rho^* \sigma_{er})$). In the derivation of Eq. (2.1) it was assumed that the influence of the erosion produced on the flow of the mixture is insignificant. This is clearly satisfied when the flow rate of the condensed phase is low ($\rho_a/\rho < 1$). For $\rho_a \geqslant \rho$ the injection of erosion products from the surface being destroyed can alter the parameters of the two-phase boundary layer. Since the flow is assumed to be inviscid in the present paper, we also neglect the influence of the erosion products. Thermal destruction of the wall material is also ignored in the model being used. Estimates show that in the investigated region of the parameters ($u_a \gtrsim 1$ km/sec) the erosion rate exceeds the rate of thermal destruction by more than an order of magnitude. The influence of the wall temperature on the erosion rate can be taken into account through the definition of the function $\sigma_{er} = \sigma_{er}(T_w)$ [13]; however, in the present paper the influence of temperature is not taken into account.

3. Comment on the Stability of the Process of Nozzle Erosion

We use the approximation of minor slippage of the condensed phase ($\varepsilon \ll 1$). Substituting Eq. (1.4) into (2.2), we obtain the equation

$$\frac{\partial y_w}{\partial t} = -D(x, y'_w)\left|\frac{\partial^2 y_w}{\partial x^2}\right|^q \frac{\partial^2 y_w}{\partial x^2}, \tag{3.1}$$

where $D(x, y'_w) = l_a^{q+1} G(1 + y_w'^2)^{1-q}$ is a nonnegative function. In analyzing Eq. (3.1) we conclude that the process of erosion with convective outward transport of particles develops like a system with negative viscosity [14]. Consequently, the system of equations (1.3), (2.2) is asymptotically unstable. The physical factors promoting the development of instability in a system of two-phase flow and a confining eroding surface are rather obvious. In accordance with Eq. (2.2), the erosion rate is the higher, the greater the collision angle, which, as follows from the approximate expression (1.4), increases with an increase in the local curvature of the wall. If a small hole forms in the erosion region, then the erosion rate increases somewhat in the vicinity of the hole, leading to an increase in the local curvature. Equation (3.1) predicts that this tendency will be strengthened. To find the most unstable mode of disturbance we must consider the system of equations (1.3), (2.2), since (3.1) is valid only for disturbances with a wavelength $\lambda \gg l_a$. Since these equations have variable coefficients, it does not seem possible to solve this problem in the general case. However, an analysis of a system describing the erosion of a thin profile [6] allows us to estimate the time of development of the instability as

$$\tau \approx (R/l_p)^q l_p/G, \tag{3.2}$$

where R is the local radius of curvature of the wall. In using the estimate (3.2) in calculations, one must consider that this may prove inapplicable in situations where nonlinear effects are important. Finally, it is interesting to note that in the sputtering of particles onto a surface the process will develop stably, since in this case an equation of the type (3.1) but with the opposite sign on the right side will also be valid.

4. Numerical Model and Some Results of the Calculations

A variety of parameters capable of affecting the erosion process were taken into account in the development of a numerical model of two-phase flow in a channel with eroding walls. In the first stage of the investigation a quasi-one-dimensional model was developed, permitting a wide range of parametric research.

To calculate the parameters of two-phase flow in an axisymmetric channel we used the Kligel–Nickerson model, described in detail in [15]. The initial profile of the nozzle was assigned by a set of seven parameters; its general form is presented in Fig. 1. The parameters ρ_a, u_a, and l_a of the condensed phase were calculated within the framework of the quasi-one-dimensional theory, and then they were used in the numerical integration of Eqs. (1.3) and (2.2). Seeking to obtain qualitative results on the development of the process of nozzle erosion, we took a flow rate $(\rho u y)_w$ equal to the average flow rate in the channel, i.e., $(\rho u y)_w = \text{const}/y_w$.

The bundle of trajectories was determined from Eq. (1.3), the local values of the angle and velocity of the collision were calculated, Eq. (2.2) was investigated over one time step, the position $y_w(x, t)$ of the solid boundary was refined, and then the bundle of trajectories was calculated for the altered conditions. The procedure was repeated the required number of times. The total integration time did not exceed the time of development of instability in the system, determined from (3.2) $(T < \tau)$. Special calculation methods are required to study the development of instability, which is a task for future research.

In Fig. 2 we present the results of a calculation of the maximum erosion depth $\Delta\bar{y}_w = [y_w(x, t) - y_w(x, 0)]_{max}/y_w(x_{max}, 0)$, normalized to the radius of the through cross section, as a function of the size of the eroding particles (curve 2). One can see that there is a particle fraction presenting the greatest danger of destruction of a nozzle with a given wall geometry. We note one peculiarity in the calculation of the region of destruction for erosion by monodisperse particles. In this case the left-hand boundary x_1 of the calculation region is a moving boundary, so that the geometry of the erosion crater to the right of the point x_1 is determined by the inclination $dy_a^*(x_1)/dx$ of the limiting trajectory. In Fig. 3 it is shown how the calculation algorithm operates with allowance for this fact. Curves 1-3 denote the position of the nozzle profile for erosion by particles with diameters of 2.0, 3.3, and 4.3 μm, respectively, and curve 4 is the position of the nozzle profile before the start of erosion. One can see that curves 1-3 are constructed from pieces of the limiting trajectories $y_a^*(x)$ (denoted by dashes) and are extended to the right of the point $x^*(t)$ by a profile $y_w(x, t)$ satisfying the condition $y_w(x^*, t) = y_a^*(x^*, t)$.

In Fig. 2 we show the dependence of the erosion rate U at x = 0.98 (indicated by an arrow in Fig. 1) on the size of the eroding particles (curve 3). It is seen that the erosion rate grows monotonically with an increase in particle diameter, which also follows directly from Eq. (1.4) for the difference in inclinations, $y_a' - y_w'$.

We note that for erosion by polydisperse particles one must use Eqs. (2.1) to calculate the resulting damage. From an analysis of the data presented in Figs. 2 and 3 it follows that in this case one must know the size distribution function of the particles. The form of the distribution curve has a more significant influence on the erosion than on the other parameters of the two-phase flow, which is consistent with the conclusions of [16].

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FLUX METHOD IN THE KINETICS OF COAGULATION

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UDC 54-138

Coagulation causes broad fluctuations in the concentration and size distribution of aerosol particles [1]. Measured particle-radius distributions are dome-shaped. The top part of the dome is usually described by the so-called log-normal distribution. The right side may descend significantly slower than the left side in accordance with a power law [2]. A power spectrum was observed for an atmospheric aerosol in [3]. Later on it was explained on the basis of a representation of constant mass flux over the particle spectrum. The form of the spectrum follows from dimensional considerations with the use of the locality hypothesis [4, 5], to within the accuracy of a coefficient. A stationary spectrum was obtained in [6] on the basis of a kinetic equation. Stationary power spectra with thermal and gravitational coagulation in different ranges of particle radius were obtained in [7] along with coefficients. It was shown in [8] that these results follow from a more general analysis of the kinetic equation with the use of the notions of fluxes of particles and mass over the spectrum. However, until now there has been no direct kinetic determination of the flux, which is important in the theory of coagulation and in certain other similar problems.

This article explicitly determines the flux of the number and volume of particles (drops) over the spectrum corresponding to the physical significance of these quantities. This

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